TECHNICAL NOTE: BUCKLING OF TIMBER SYSTEMS

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When determining the slenderness coefficient for lateral buckling under compression, clause 3.3.2 of AS 1720.1 provides the effective length factor g_{13} for columns without intermediate lateral restraint.

This technical note provides additional effective length factors for common structural systems used in timber design. The information provided is based on the German National Annex to Eurocode 5 and relevant literature.

1 STIFFNESS VALUES

When determining the effective length factors to Section 2 of this note, the stiffness properties of the member or its connection shall be reduced by the capacity reduction factor.

$$E' = \frac{E}{\phi} \tag{1}$$

$$G' = \frac{G}{\phi} \tag{2}$$

$$K'_\theta = \frac{K_\theta}{\phi} \tag{3}$$

where

E' = reduced modulus of elasticity

G' = reduced shear modulus

K'_θ = reduced rotational stiffness of the semi-rigid connection (*force x length / angle*)

Ø = capacity reduction factor

2 EFFECTIVE LENGTH FACTORS FOR TIMBER SYSTEMS

In addition to the effective length factors g_{13} in Table 3.2 of AS 1720.1, the values in Table 1 can be used to determine the slenderness coefficients to Clause 3.3.2.

Note that the effective length factor for system 3 is valid for both two and three hinged arches.

For tapered members, the section properties measured at 0.65 of the member length from the smaller end shall be used.

When verifying a compression member under buckling where the axial load varies along the length, the largest value for axial compression shall be used.

3 ADDITIONAL MOMENT IN A SEMI-RIGID CONNECTION

The additional moment in a semi-rigid connection due to the buckling restraint in systems 1, 2 and 4 as per Table 1 can be calculated as

$$M^* = N^* \frac{d}{6}\left(\frac{1}{k_{12}} - 1\right) \tag{14}$$

where

d = depth of the member

k_{12} = stability factor of the member

For system 4, the moment *M** is to be taken as the larger between the moments generated by the column and the rafter.

4 EFFECTIVE LENGTH FACTOR INCLUDING SHEAR STIFFNESS

When determining the slenderness coefficient for buckling with Equations 3.3(6) or 3.3(9) of AS 1720.1, the length *L* shall additionally be multiplied by the factor g_{14} if the shear stiffness of the member needs to be considered.

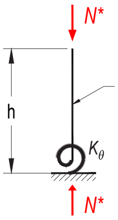
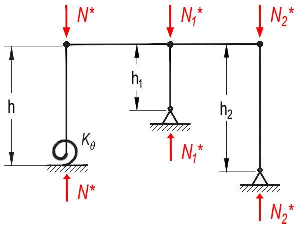
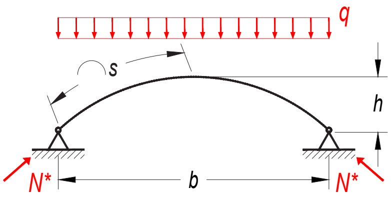
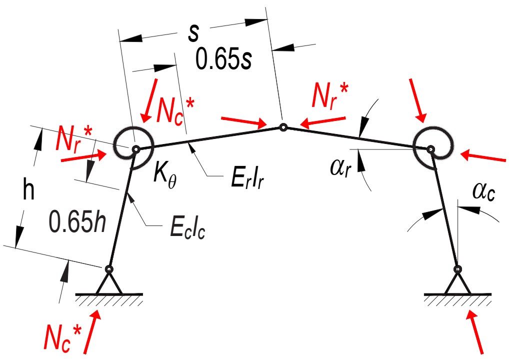
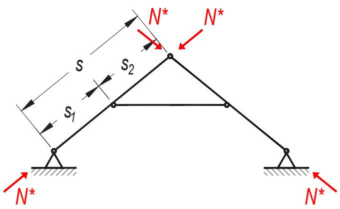
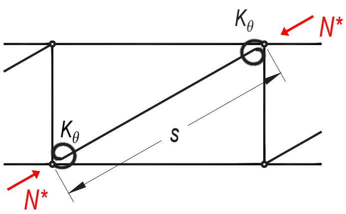
$$g_{14} = \sqrt{1 + \frac{E'\, I\, \pi^2}{(g_{13}\, L)^2\, S}} \tag{15}$$

where

I = moment of inertia of the member

L = length between points of effectively rigid restraint

Table 1: Effective length factors g_{13} for timber systems

<table>
<tr><th colspan="2">System</th><th>Effective length factor (g_{13})</th><th></th></tr>
<tr><td>1</td><td>
</td><td>$$g_{13} = \sqrt{4 + \frac{\pi^2 E' I}{h K'_\theta}}$$</td><td>(4)</td></tr>
<tr><td>2</td><td>
</td><td>For the member with the semi-rigid constraint
$$g_{13} = \sqrt{\left(4 + \frac{\pi^2 E' I}{h K'_\theta}\right)(1 + \alpha)}$$
where $\alpha = \frac{h}{N^*} \sum \frac{N_i^*}{h_i}$
and $L = h$</td><td>(5)
(6)</td></tr>
<tr><td>3</td><td>
</td><td>for $0.15 \le \frac{h}{b} \le 0.5$
$g_{13} = 1.25$
where $L = s$ for asymmetric buckling</td><td>(7)</td></tr>
<tr><td>4</td><td>
</td><td>Column
$$g_{13,c} = \sqrt{4 + \frac{\pi^2 E'_c I_c}{h}\left(\frac{1}{K'_\theta} + \frac{s}{3 E'_r I_r}\right) + \frac{E'_c I_c N_r^* s^2}{E'_r I_r N_c^* h^2}}$$
where $L = h$ and $\alpha_c \le 15°$ for asymmetric buckling
Rafter
$$g_{13,r} = g_{13,c} \sqrt{\frac{E'_r I_r N_c^*}{E'_c I_c N_r^*} \frac{h}{s}}$$
where $L = s$ and $\alpha_r \le 20°$ for asymmetric buckling</td><td>(8)
(9)</td></tr>
<tr><td>5</td><td>
</td><td>For $s_1 < 0.7s$ for asymmetric buckling
$g_{13}=0.8$
For $s_1 \ge 0.7s$ for asymmetric buckling
$g_{13}=1.0$
where $L = s$</td><td>(10)
(11)</td></tr>
<tr><td>6</td><td>
</td><td>For hinged connection ($K_\theta \approx 0$)
$g_{13}=1.0$
For semi-rigid connection ($K_\theta \gg 0$)
$g_{13}=0.8$</td><td>(12)
(13)</td></tr>
</table>

The shear stiffness S for rectangular sections is

$$S = \frac{G' A}{1.2} \tag{16}$$

where

G' = reduced shear modulus

A = cross sectional area of the cross section

The shear stiffness S for I-beams is

$$S = G'_w \, b_w \, h_{eff}$$

where

G'_w = reduced shear modulus of web

b_w = width of the web

h_{eff} = effective height of the web (distance between centres of gravity of flanges)

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The buckling strength of some members might be reduced by shear deformation, especially for stubby members with low slenderness ratios. In such cases the length L shall additionally be multiplied by the factor g_{14} calculated with Equation (15). This modified length shall be used for calculating the slenderness coefficient for buckling with Equations 3.3(6) or 3.3(9) of AS 1720.1.

Typically, the shear stiffness has little effect on the buckling strength, as stubby members are not prone to buckle.

5 FURTHER READING

AS 1720.1:2010. Timber Structures – Part 1: Design methods. Standards Australia

DIN EN 1995-1-1/NA:2013-08 National Annex – Nationally determined parameters – Eurocode 5: Design of timber structures – Part 1-1: General – Common rules for construction. DIN Deutsches Institut für Normung e.V.