DESIGN OF RECTANGULAR TIMBER BEAMS FOR TORSION

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NZS 3603 and its replacement NZS AS 1720.1 do not contain provisions for the design of beams subjected to torsion. While not common in standard timber design, this situation does occur, and the following verification method is presented to enable a designer to check a rectangular beam subjected to this type of load.

This follows the current methods contained within Eurocode 5 Amendment A2. Formatting has been made to be consistent with DZ NZS AS 1720.1/V6.0 (here cited as NZS AS 1720.1 for clarity).

1 TORSIONAL STRENGTH

The design capacity in torsion ($M_{d,t}$) of a rectangular member, for the strength limit state, shall satisfy the following:

$$M_{d,t} \geq M_t^* \qquad (1)$$

Where:

$$M_{d,t} = \varphi k_1 k_4 k_6 k_{30} k_{31} d b^2 f_s' \qquad (2)$$

And

M_t^* = design action effect in torsion

$M_{d,t}$= design capacity in torsion (see Clause 1.4.2.2 NZS AS 1720.1)

φ = capacity factor (see Clause 2.3 NZS AS 1720.1)

k_1, k_4, k_6 = modification factors given in Section 2 of NZS AS 1720.1

k_{30} = modification to relate torsional shear strength to direct shear strength (see Section 1.1)

k_{31} = modification factor depending on breadth to depth ratio (see Section 1.2)

d = depth of the structural member

b = breadth of the structural member

f_s' = characteristic value in shear

1.1 Torsional shear strength to direct shear strength modification factor for torsion

The torsional shear strength to direct shear strength factor (k_{30}) for rectangular beams shall be given by the following:

$$k_{30} = min\begin{pmatrix} 1 + 0.05(\frac{d}{b}) \\ 1.3 \end{pmatrix} \qquad (3)$$

1.2 Breadth to depth ratio modification factor for torsion

The breadth to depth ratio modification factor for torsion in rectangular members (k_{31}) is given in Table 1. For intermediate values, the factor shall be obtained by linear interpolation.

Table 1: Breadth to depth ratio modification factor for torsion in rectangular members.

d/b	k_{31}
1	0.208
1.5	0.231
1.75	0.239
2	0.246
2.5	0.258
3.0	0.267
4.0	0.282
6.0	0.299
8.0	0.307
10.0	0.313

2 COMBINED SHEAR AND TORSION

Rectangular members subjected to combined flexural shear and torsion shall be proportioned so that:

$$\frac{M_t^*}{M_{d,t}} + \left(\frac{V_x^*}{V_{d,x}}\right)^2 + \left(\frac{V_y^*}{V_{d,y}}\right)^2 \leq 1 \qquad (4)$$

C1 TORSIONAL STRENGTH

In accordance with Euler-Bernoulli theorem the maximum torsional shear stress in an elastic rectangular section can be calculated by:

$$\tau_{tor} = \frac{M_t}{\alpha d b^2} \qquad (5)$$

Where:

M_t = applied torsional moment

α = a torsional constant for a given ratio of breadth and depth of a closed rectangular section (see C1.2).

For the purposes of this paper and constancy with NZS AS 1720.1 this constant has been named k_{31}.

Rearranging the above relationship provides the basis of Equation 1.

C1.1 Shear strength to direct shear strength modification factor for torsion

Equation 5 calculates the maximum torsional stress of the section, however, it is more common for the shear in beams strength to be available to designers and therefore the conversion factor is used. The values provided by Equation 3 are lower than those originally used in Eurocode 5 as these have been found to be potentially un-conservative (J. Porteous and A. Kermani, Structural Timber Design to Eurocode 5, 2nd ed., Wiley Blackwell, 2013). Eurocode 5 Amendment A2 reflects these lowered values.

C1.2 Breadth to depth ratio modification factor for torsion

The factor k_{31} relates to the use of the membrane analogy for a closed rectangular section. The full derivation can be found in S. P. Timoshenko and J. N. Goodier, Theorey of Elasticity, 3rd ed., McGraw-Hill, New York, 1969.

C2 COMBINED SHEAR AND TORSION

There is only limited available information regarding the combination of torsion and shear in beams and knowledge in this area remains limited. However, as these stresses do interact, this check is required.